Crack Growth and Failure of Aluminum Plate under In-Plane Shear

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Experimental studies have been conducted to investigate the physical and engineering aspects of combined Mode 1 and Mode 2 fracture behavior. Test panels containing fatigue cracks oriented in various directions were fabricated from 2024-T3 and 7075-T7651 aluminum alloy plate. Shear loads were applied at the edges of the test panel. The predicted crack growth directions agree reasonably well with the actual crack growth angles measured from the broken test panels. An interaction curve constructed from the present shear panel test results is compared with the tension specimen test data of other investigators and to the behavior predicted by the Griffith-Irwin theory of fracture. In consequence of these comparisons, it is speculated that the K_1 and K_2 interaction behavior is somewhat dependent upon the applied loading conditions. However, more data are required before a refined semiempirical criterion for fracture under combined loading conditions can be developed.

Nomenclature

f_g = applied gross area tensile stress
f_s = applied gross area shear stress
F_s = gross area shear stress at failure of the panel
F_{ty} = material tensile yield strength
K_1 = opening mode stress intensity factor, the state of stress at the crack tip (degree of triaxiality) is undefined, i.e., it could be either plane stress or plane strain
K_2 = sliding mode stress intensity factor, the state of stress at crack tip is also undefined
K_{1c} = the critical value of K_1
K_{2c} = the critical value of K_2
K_C = the critical stress intensity factor for plane stress or plane stress mixed with plane strain, opening mode
l = crack length
l_f = final crack length
l_0 = initial crack length
Δl = $l_f - l_0$
t = thickness of the panel
W = panel width

Introduction

CURRENT analysis methods for designing fail-safe aircraft structures have been limited to the analysis of cracked plates or sheets, either stiffened or unstiffened, under tension load. It has been assumed that the loading direction is always perpendicular to the crack and that the panel is subjected to uniaxial extensional load only.

However, there are three basic modes of crack tip displacements: Crack Opening Mode (called Mode 1), Crack Sliding Mode (Mode 2), and the Anti-Plane Shearing Mode (Mode 3). Examples of a flat cracked plate subjected to in-plane loading where the third mode of crack tip displacement is absent are given in Figs. 1 and 2. These sketches show that the crack is at an angle to the principal tension or shear stress. A more general case is the superposition of Figs. 1 and 2. In either case, the resolved tension stress perpendicular to the crack tends to open the crack (Mode 1). The resolved shear stress parallel to the crack tends to slide the crack (Mode 2). In other words, the crack tip displacement, for the cases given in Figs. 1 and 2, will either be Mode 1, Mode 2, or combined Modes 1 and 2 depending on the type of loading and the loading direction.

The failure criterion for these stress field interactions is

$$(K_1/K_{1c})^u + (K_2/K_{2c})^v = 1 \tag{1}$$

where K_1 and K_2 are the applid Mode 1 and Mode 2 stress intensities and K_{1c} and K_{2c} are the critical values for failure of the sheet or plate under pure Mode 1 or pure Mode 2. These critical values depend upon the plate thickness as well as the

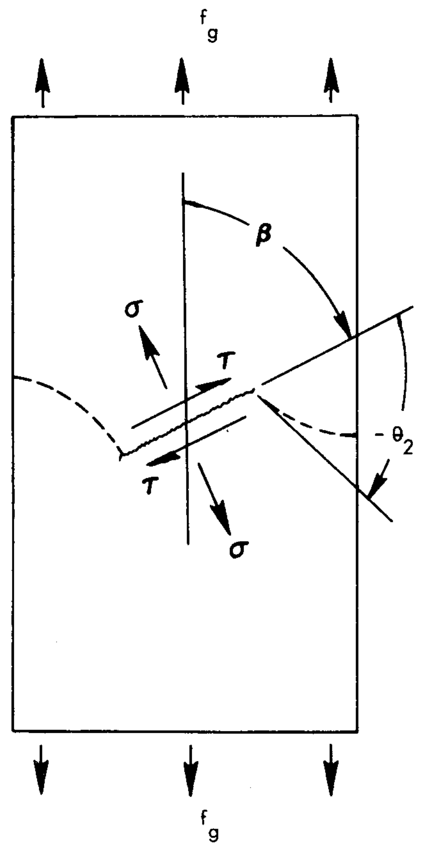

Fig. 1 Stress components on a crack subjected to tension load.

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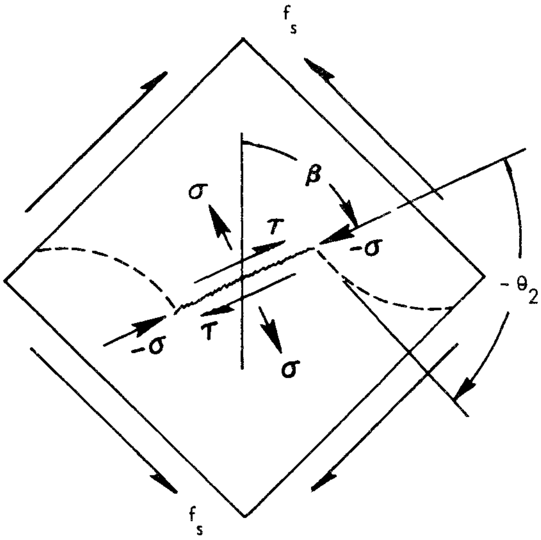

Fig. 2 Stress components on a crack subjected to shear load.

material. Both Erdogan and Sih,[1] and Irwin,[2] have postulated that the K_1 and K_2 interaction behavior of Eq. (1) would be an ellipse, i.e., $u = v = 2$. In the present study, u and v are treated as experimentally determined exponents. An experimental program was conducted to generate data for 2024-T3 and 7075-T7651 high strength clad aluminum plate materials. The characteristics of crack growth under shear load were studied. An engineering failure criterion suitable to all practical purposes was explored.

Tests

Using the specimen configuration shown in Fig. 1 it would not be possible to achieve a pure Mode 2 condition and directly obtain the K_{2c} value. On the other hand the specimen configuration of Fig. 2 will permit any combination of K_1 and K_2 including the "pure" conditions simply by choice of the crack orientation; therefore, this panel configuration was selected for the test program. One dummy panel (no crack) and seven panels with cracks were fabricated. The materials were 0.6-in. thick 7075-T7651 high strength clad plate machined on one side to a thickness of 0.3-in. and as received 2024-T3 bare $\frac{5}{16}$-in. thick plate.

Prior to final fabrication of the panels, oversized panels of Fig. 1 configuration were cut from a large plate. On each of these panels, a 4-in. saw cut was made at the center parallel to the width direction and perpendicular to the plate rolling direction. Tension-tension cyclic stresses, at a very low stress level, were applied to the panel to generate a fatigue crack at the ends of the saw cut. Fatigue cycling was stopped when the crack reached a total length of 6 in. Then the oversized panel was removed from the machine and fabricated to final dimensions with the crack oriented in the desired direction so that different ratios of K_1 to K_2 could be obtained. The panel configurations are listed in Table 1. The panels were installed in a 22.5-in. by 22.5-in. picture frame jig with all four edges bolted to the frame. Tension loads were applied through a pair of pins located diagonally at two corners of the frame so that essentially uniform shear flow was produced along the edges of the panel. In addition to the load-deflection records, motion pictures were taken during the test run.

The Crack Tip Stress Intensity

In a shear panel (Fig. 2) pure Mode 2 occurs when the crack is parallel to any edge of the panel. Pure Mode 1 is established when the crack is diagonal. A combined Mode 1 and Mode 2 condition is obtained when $45° \leq \beta \leq 90°$. Note that in all cases except for pure Mode 2 there is a compressive stress component parallel to the crack. Since this compressive stress does not contribute to the crack opening or crack sliding movement, it is assumed that the magnitude of the crack tip stress intensity will not be altered due to the presence of this compressive stress

Table 1 Shear panel configurations and crack growth behavior

Alloy	Specimen no.	β^c (degree)	Crack tip stress	θ_2 (deg.) Predicted[a]	Actual[b]	Remarks
7075-T7651 high strength clad, machined one side from 0.6 in. to 0.3 in. thickness	SP-3	45	Pure shear	−70.5	−72/75	$l_0 = 6$ in., no crack growth in the new direction
	SP-4	60	Combined stress	−60	−60/60	$l_0 = 6$ in., no clear indication of crack growth (may be $\frac{1}{4}$ in. growth on each side in the new direction)
	SP-5	75	Combined stress	−43.2	−27/32	$l_0 = 6$ in., crack growth $\frac{3}{16}$ in. on one side, $\frac{13}{16}$ in. on the other side, both in the new direction
	SP-6	90	Tension	0	0	$l_0 = 6.25$ in., $\frac{1}{2}$ in. tunneling on each side, $l_f = 8.25$ in. determined by film record
2024-T3 Bare $\frac{5}{16}$ in. thick as received	SP-13	45	Pure shear	−70.5	−79/80	$l_0 = 6$ in., no crack growth
	SP-15	75	Combined stress	−43.2	−25/32	$l_0 = 6$ in., app. $\frac{1}{2}$ in. tunneling on each side along new direction
	SP-16	90	Tension	0	0	$l_0 = 6$ in., $\frac{3}{4}$ in. tunneling on each side $l_f = 10.0$ in. determined by film record[d]

[a] Equation (7).
[b] Numbers refer to each side of the crack tip.
[c] Crack always perpendicular to grain.
[d] Slow crack growth all the way across the panel under constant load (at max. load), rapid fracture did not occur.

component. Therefore, this stress component is not included in the calculation of the stress intensity factor.

The Mode 1 and Mode 2 stress intensity factor for an inclined crack under shear load is conveniently expressed in terms of β, the angle between the crack and the applied tensile load. Thus

$$K_1 = -f_s(\pi l/2)^{1/2} \cdot \cos 2\beta \cdot \phi_1 \tag{2a}$$

and

$$K_2 = f_s(\pi l/2)^{1/2} \cdot \sin 2\beta \cdot \phi_2 \tag{2b}$$

with $45° \leqq \beta \leqq 90°$ which gives

$$K_1 = 0 \quad \text{for} \quad \beta = 45°$$

and

$$K_2 = 0 \quad \text{for} \quad \beta = 90°$$

Here, f_s is the applied shear stress, l the crack length. Due to the loading conditions and the fact that the crack tips are bounded by two inclined edges of the panel, the geometry factors ϕ_1 and ϕ_2 are unknown. However, in the center region of the plate (containing a short crack), the effect of ϕ_1 and ϕ_2 is estimated to be within 5% (i.e., $\phi_1 \simeq \phi_2 \simeq 1.05$).

Usually the effective crack length is considered to be $(2a + 2r_p)$ where r_p is the radius of the crack tip plastic zone and a is the half-crack length. For plane stress[3,4]

$$r_p = (1/2\pi F_{ty}{}^2)[K_1{}^2 + 3K_2{}^2] \tag{3}$$

where F_{ty} is the tensile yield strength of the material.

Substituting Eqs. (2a) and (2b) into Eq. (3), and setting ϕ_1 and ϕ_2 equal to unity to represent a very small a/W ratio, we obtain

$$r_p = (f_s/F_{ty})^2 \cdot [\tfrac{1}{2} + \sin^2 2\beta] \cdot a \tag{4}$$

The Direction of Crack Growth

Using the cylindrical system, the stress field in the vicinity of the crack tip is given by[5]

$$\begin{aligned}
\sigma_r &= \frac{1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[K_1\left(1+\sin^2\frac{\theta}{2}\right) + K_2\left(\frac{3}{2}\sin\theta - 2\tan\frac{\theta}{2}\right)\right] \\
\sigma_\theta &= \frac{1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[K_1\cos^2\frac{\theta}{2} - \frac{3}{2}K_2\sin\theta\right] \\
\tau_{r\theta} &= \frac{1}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}[K_1\sin\theta + K_2(3\cos\theta - 1)]
\end{aligned} \tag{5}$$

In the maximum stress criterion for brittle fracture suggested by Erdogan and Sih,[1] the crack is assumed to move along a path normal to the direction of greatest tension. In this case, the component of shear stress on the line of expected extension of the crack is zero, i.e., $\tau_{r\theta}$ is set equal to zero. Then

$$\cos\frac{\theta}{2}[K_1\sin\theta + K_2(3\cos\theta - 1)] = 0$$

If θ_2 is defined as the angle at which the circumferential stress, σ_θ, is a maximum, then the preceding equation yields

$$K_1\sin\theta_2 + K_2(3\cos\theta_2 - 1) = 0 \tag{6}$$

Clearly the solution for θ_2 depends on the relative magnitudes of K_1 and K_2. For the case shown in Fig. 2, an inclined crack in a shear stress field, substitute Eqs. (2a) and (2b) into Eq. (6). With $45° \leqq \beta \leqq 90°$, and $\phi_1 = \phi_2 = 1.0$, we have

$$-\cos 2\beta \cdot \sin\theta_2 + \sin 2\beta \cdot (3\cos\theta_2 - 1) = 0 \tag{7}$$

which predicts

$$\begin{aligned}
\theta_2 = -70.5° &\quad \text{for} \quad \beta = 45° \\
-60° &\qquad\qquad 60° \\
-43.2° &\qquad\qquad 75° \\
0° &\qquad\qquad 90°
\end{aligned}$$

The minus sign for θ_2 implies that the crack would start to grow in the directions marked by the curved lines at the crack tips in Fig. 2.

Results and Discussion

A preliminary check was made to verify that the loading fixture would produce uniform shear throughout an uncracked panel. A 10-in. by 10-in. photoelastic coating was placed on one side and six rosette gages were placed on the opposite side of the panel. Incremental stepwise loads of 25,000 lb, up to 200,000 lb, were applied to the panel. Comparison of the calculated values with the experimentally determined values revealed that the strain gage data was within 0% and +5% of the theoretical value, and the photoelastic data was within ±3% of the theoretical value. However, the photoelastic stresses were only measured for loads up to 125,000 lb. In this load range, the variations in the strain gage readings were within 3.5%. Panel buckling was not observed over the entire range of the applied loads.

The failure loads for the crack panels are reported in Table 2. The load-displacement (machine cross head movement) trace records are shown in Fig. 3. It is seen that the responses of the 7075-T7651 aluminum panels were linear throughout the entire load range for each test. For the 2024-T3 aluminum panels, the gradual curvature at the end of the load displacement curves indicates that crack-tip plastic deformation occurred during the crack extension process.

The failed panels were examined visually to determine the direction of crack propagation and the amount of slow stable crack growth prior to fracture of the panel. The measurements were made at, or as close as possible to, the midthickness of the panel. The results are tabulated in Table 1. Generally, there was a negligible amount of slow stable crack growth in the panels loaded in pure Mode 2. The amount of crack growth increased as the initial crack orientation was rotated towards the pure Mode 1 position. The amount of slow stable tear for the pure Mode 1 tests in the shear panels was approximately 1 in. on each side of the crack (30% of the initial crack length) for the 7075-T7651 alloy. The amount of slow stable tear for the 2024-T3 alloy was as much as 2 in. on each side of the crack or 65% of the initial crack length.

Table 2 Stress intensity components for the shear panels

Specimen no.	P (*kip*)	F_s (*ksi*)[a]	K_1 *ksi* (in.)$^{1/2}$	K_2 *ksi* (in.)$^{1/2}$	K_{1c} *ksi* (in.)$^{1/2}$	K_{2c} *ksi* (in.)$^{1/2}$	$K_c{}^b$ *ksi* (in.)$^{1/2}$
SP-3	242.75	25.4	0	81.2	...	81.2	...
SP-4	189.25	19.8	27.6	56.0	...	...	...
SP-5	189	19.8	54.6	31.8	...	...	...
SP-6	201	21.05	68.7	0	68.7	...	...
							81.2
SP-13	215.5	21.7	0	68.0	...	68.0	...
SP-15	202	20.4	50.6	38.2	...	...	...
SP-16	208	21.0	67.5	0	67.5	...	...
							90.5

[a] $F_s = P/22.5(2)^{1/2} \cdot t$.
[b] Center-cracked panel, Ref. 8.

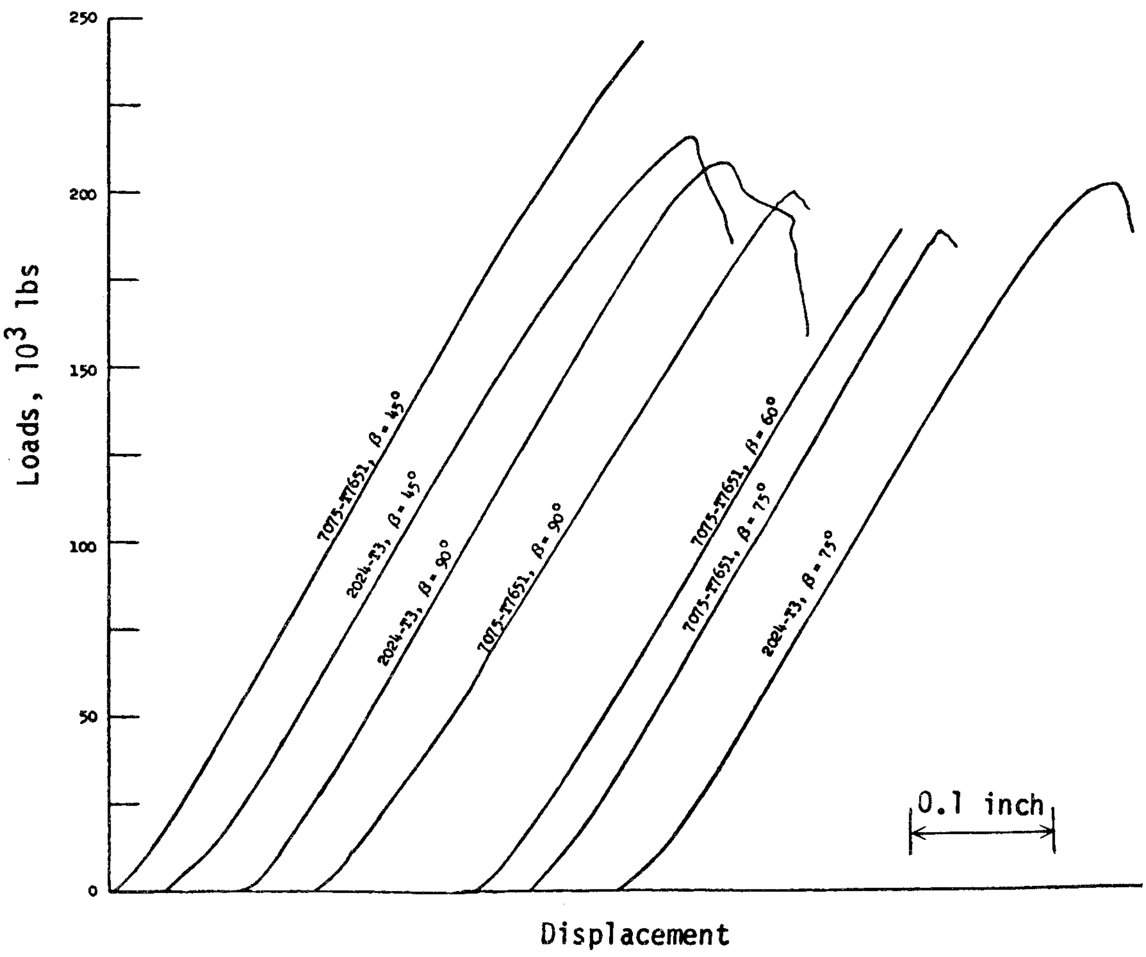

Fig. 3 Load-displacement traces.

The amount of slow stable tear (Δl) exhibited by a given material during a test will depend on whether the test configuration is stable or unstable. The remote loading condition is less stable than the configuration having loads applied close to the crack (e.g., wedge forces). For the panels subjected to pure shear ($\beta = 45°$), the loads around the crack are rather remote. As β increases, the crack at the center of the panel (remote from the edges of the frame) started to grow from a less stable position to a more stable position (toward the area bounded by the panel edges). In other words, if these panels were sufficiently large, the slow stable crack growth behavior might have been similar in all the cases.

The predicted and the measured crack growth angles, θ_2, are plotted in Fig. 4 vs K_1/K_2. Also shown in Fig. 4 are the center-cracked panel test results of Pook,[4] and the crack growth angles predicted by the "energy theory" of Palaniswamy and Knauss.[6] It is seen that for $K_1 < K_2$, the measured θ_2 exhibited a close agreement with the predicted θ_2 values. However, as pure Mode 1 is approached, the measured angles departed from the predictions. In the derivation of Eq. (7) it was assumed that the panels were infinitely wide and the material behavior was elastic. In the 22.5-in. wide shear panels tested, the direction of crack growth might have been significantly affected by the finite boundary of the test panels, resulting in the deviation from theory that was observed for large β.

Data Correlation

To develop a design criterion such as that given in Eq. (1), a practical choice of fracture parameters is essential. To maximize the applicability of linear elastic fracture mechanics theory, an ideal case is that of a crack plate made of high strength, low toughness material, loaded under a less stable condition. Under these circumstances, the final crack length, l_f, at failure would be approximately equal to the initial crack length, l_0. Also, the size of r_p would be negligibly small.

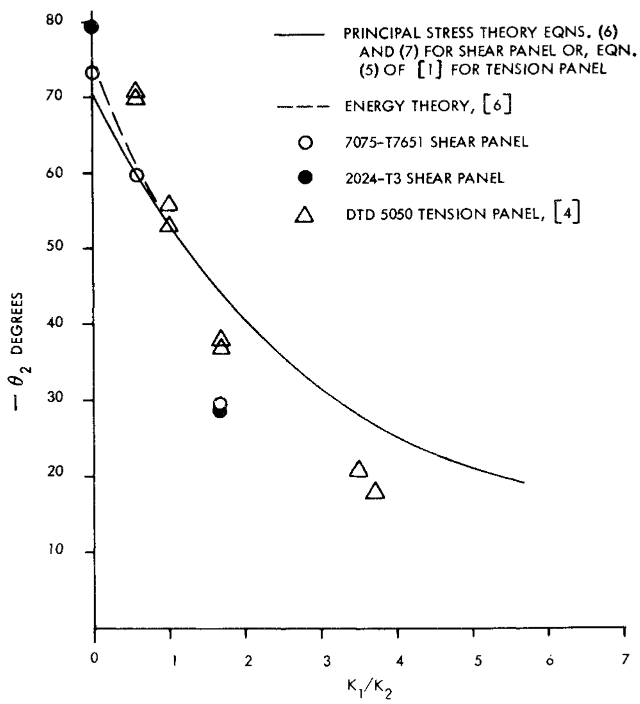

Fig. 4 Directions of crack growth.

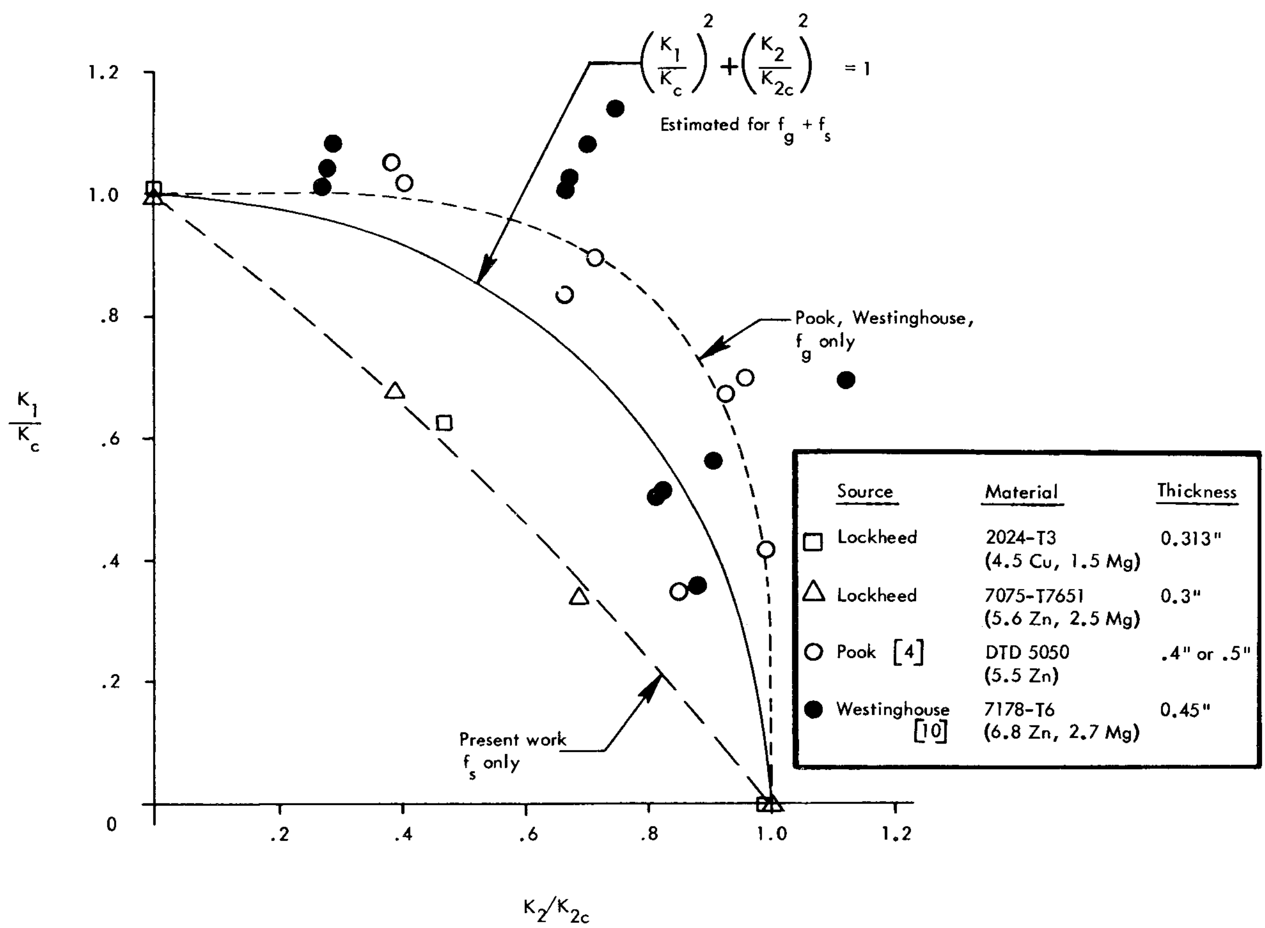

Fig. 5 Behavior of K_1 and K_2 interactions.

In the present shear panel tests, the fracture processes were very complicated. As would be observed in Eqs. (3) and (4), the size of r_p increases as the magnitude of K_2 increases. Contrarily, the amount of slow stable tear (Δl) increases as β increases. Also, the crack extends in a new orientation, θ_2. Consequently, determination of the K parameters in Eq. (1) by using Eqs. (2–4) would be extremely difficult. A fictitious parameter "K (apparent)," which is defined to be a function of initial crack length and maximum load without plastic zone corrections, may be useful (e.g., see Ref. 7). Adopting this parameter, the K_1 and K_2 components for the test panels were computed and listed in Table 2. Previously determined K_c values (also calculated using initial crack length and maximum load)[8] are also shown in Table 2 for comparison. Note the K_c value for the 7075-T7651 alloy was developed from the same heat and the same thickness as for the shear panel. It is seen that the critical stress intensity factors for pure Mode 1, "K_{1c}(apparent)," for the shear panels were considerably lower than the "K_c(apparent)" values which were developed from testing of center-cracked panels having approximately the same width (24 in.).

As previously noted, the shear panels exhibited a large amount of slow stable tear. However, although not shown here, the amount of slow stable tear for the same material and thickness in a center-cracked panel test would be only 5%–10% for 7075-T7651 and 10%–20% for the 2024-T3 alloy. These differences were probably due to the effects of panel geometries and loading conditions as previously discussed. Additionally, the combinations of the shear panel boundary conditions and the compressive stresses parallel to the crack surfaces, which caused excessive local buckling (as compared to the tension panel tests), might have altered the crack growth resitance behavior of the shear panel. (See Ref. 9 for general discussions of the crack growth resitance behavior.)

Since fracture toughness data are usually obtained by testing of center-cracked panels, failure criteria are established based on this type of data. More specifically, the "K_c(apparent)" parameter is acceptable for design purposes because it only requires knowing the ultimate stress that a damaged structural member can sustain. Therefore, despite the complexities discussed previously, an engineering criterion for mixed mode failure can be developed using "K_c(apparent)" as a baseline fracture index. The calculated K_1/K_c and K_2/K_{2c} ratios for each panel are plotted in Fig. 5. Here again, all K parameters are "K (apparent)." The u and v exponents can be determined by fitting an interaction curve through the data points ($u = v = 1.08$ in this case).

Data developed by Pook[4] and Wilson et al.[10] on aluminum alloys similar to 7075 aluminum are also plotted in Fig. 5 to aid in evaluation of the K_1 and K_2 interaction behavior. Their data were developed by testing subsized specimens with inclined cracks (2-in.-4-in. wide, Fig. 1 configuration). One disadvantage of this type of specimen is that pure Mode 2 failure cannot be obtained and extrapolated K_{2c} values had to be used. Another disadvantage is the small specimen size, which reduces the applicability of fracture mechanics theory. Nevertheless it is clear that the interaction behavior of the specimens tested in tension is distinguishable from those tested in shear.

Although it is not the intention of the present work to verify the theoretical exponents of Eq. (1), the interaction curve having $u = v = 2.0$ fits between the two groups of data points. This observation may indicate that the tension panel (applied f_g only) and the shear panel (applied f_s only) are two extreme cases. The Mode 1 and Mode 2 interaction behavior is not solely a function of the relative magnitudes of K_1 and K_2 but is also dependent upon the loading conditions. It is also anticipated that the theoretical relationship would be a suitable criteria for the general cases such as that of a large structural plate under combined tension and shear. However, more data and further studies are required before more refined relationships for various degrees of combined tension and shear loads can be developed.

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Statistical Identification of Structures

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A method is formulated for systematically using experimental measurements of the natural frequencies and mode shapes of a structure to modify stiffness and mass characteristics of a finite element model. Throughout the modification process, which does not require complete data, the finite element model remains consistent. An additional feature is that the engineer's confidence in the modeling of the various finite elements is quantified and incorporated into the revision procedure. Examples demonstrate the convergence and versatility of the method.

Introduction

A PRINCIPAL goal of the structural analyst is to formulate an analytical model of a structure which can be verified by actual test. Frequently, this model does not initially produce mode shapes and natural frequencies which concur with test results, and consequently an iterative cycle must be introduced to adjust the analytical model until the analysis and test results agree. The adjustment procedure is difficult and cannot be done adequately without the computer because of the large volume of computation necessary. In recent years a number of procedures have been developed, but none has received general acceptance although many have been successful for specific applications.[1]

Of the papers written on system identification, only a portion have presented methods which maintain the full order and physical significance of the mass and stiffness matrices of the structure.[2–6] Of these methods, several attempted to construct stiffness matrices from incomplete modal data and the remaining applied least squares techniques to obtain the desired property values. The use of least squares generally requires an overdetermined solution and consequently the amount of data obtained in the test must exceed in number the parameters in the structure which can be identified. As a result, the utilization of as much data as possible is very important. If only frequency is used, the number of identifiable parameters is severely limited and hence, if at all possible, measured modal deflections should be included in the procedure.

Data and modeling accuracies are both important considerations in the identification process. Certainly the accuracy of modal deflection measurements is not as good as the accuracy of frequency measurements. Therefore any procedure using least squares should include data accuracy in the weighting procedure. Modeling accuracy is more intuitive but still significant. In the development of the stiffness matrix, structural elements are included which can be modeled with varying confidence. Bending stiffnesses of uniform beams, for example, can be estimated much more accurately than bending stiffnesses for conical shells. Consequently those elements in the stiffness matrix relating to the beam have much less uncertainty associated with them than the stiffness elements relating to the conical shell. It is desirable for the method to be able to handle this variance of confidence.

The objective, therefore, of the work presented here is to propose a general method which will identify a finite element model of a structure capable of providing modal characteristics which are consistent with those measured in test. The method

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